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THE REACTION OF STRATIFIED ROTATING MEDIA TO LOCAL THERMAL EFFECTS*

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The flows which arise due to the effect of local heat sources in a stratified liquid rotating around its vertical axis are studied in the Boussinesq approximation using the method of integral transformations. Both the stationary case and the non-stationary case, which corresponds to an instantaneous heat pulse, are studied. Polyhelical convection, the structure of which is determined by the relationship between the stratification and rotational parameters as well as by the values of the dissipative parameters of the mixture, is investigated.

1. Formulation of the problem. Convective flows are described using the example when stable stratification in space, where a local source of heat acts with a specified intensity $Q(r, z, t)$, is created by changes in the concentration of a dissolved salt /1/ or changes in the concentration of suspended particles throughout the height of the liquid. The coefficients of kinematic viscosity ν, thermal diffusivity $\varkappa$, and diffusion D of a salt are assumed to be non-zero and, in general, have different values.

We shall describe the motion of a viscous thermally conducting medium which rotates around the vertical axis by means of a linear system of Navier-Stokes' equations in the Boussinesq approximation when there is axial symmetry (the origin of the rotating coordinate system is chosen to be at the point where the source acts. The angular velocity vector Ω is directed along the z-axis)

$$\frac{\partial v_r}{\partial t} = -\rho^{-1}\frac{\partial p}{\partial r} + \nu\Delta v_r + f_0 v_\varphi \tag{1.1}$$

$$\frac{\partial v_z}{\partial t} = -\rho^{-1}\frac{\partial p}{\partial z} + \nu(\Delta + r^{-2})v_z + (\beta T' - c')g$$

$$\frac{\partial v_\varphi}{\partial t} = \nu\Delta v_\varphi - f_0 v_r, \quad \frac{\partial(rv_r)}{\partial r} + \frac{\partial(rv_z)}{\partial z} = 0$$

$$\frac{\partial T'}{\partial t} = \varkappa(\Delta + r^{-2})T' + \frac{Q(r, z, t)}{\rho c_v}, \quad \frac{\partial c'}{\partial t} = D(\Delta + r^{-2})c' + \Gamma v_z$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - r^{-2} + \frac{\partial^2}{\partial z^2}, \quad \Gamma = -\frac{\partial c_0}{\partial z} = \text{const} > 0$$

Here v_r, v_z and v_φ are the radial, vertical and azimuthal components of the velocity of the medium, p is the pressure in the medium after the hydrostatic pressure has been subtracted, ρ is the density of the medium, g is the acceleration due to gravity, β is the coefficient of thermal expansion of the medium, $T' = T - T_0$, T is the temperature, T_0 is the temperature of the unperturbed medium, c_v is the specific heat capacity of the medium at constant volume, $c' = c - c_0$, c is the ratio of the density of the salt to the density of the liquid, $c_0(z)$ is the value of c in the unperturbed medium, and $f_0 = 2\Omega$ is the value of the Coriolis parameter.

2. Convection when $\nu = \varkappa = D$. Let us introduce the Stokes' stream function ψ and the azimuthal component of the vector potential B using the formulae

$$v_r = -\frac{1}{r}\frac{\partial\psi}{\partial z}, \quad v_z = \frac{1}{r}\frac{\partial\psi}{\partial r}, \quad B = r^{-1}\psi \tag{2.1}$$

If the potential B is known, v_φ can be determined from the third equation of system (1.1). The flow is of a local nature and decays at infinity. In describing flows at distance which significantly exceed the characteristic dimension of the region of heat evolution, the quantity $Q(r, z, t)$ in (1.1) may be replaced by $Q_0\delta(t)\delta(z)\delta(r)$. Under these conditions the system of Eqs.(1.1), taking (2.1) into account, together with the boundary conditions leads to the problem

$$\left[\left(\frac{\partial}{\partial t} - \nu\Delta\right)^2\Delta + \omega_0^2\left(\Delta - \frac{\partial^2}{\partial z^2}\right) + f_0^2\frac{\partial^2}{\partial z^2}\right]B = M\delta(t)\delta(z)\frac{\partial}{\partial r}\delta(r) \tag{2.2}$$

$$r = 0, \ \infty, \quad z = \pm\infty, \quad B = 0, \quad \omega_0^2 = \Gamma g; \quad M = (\rho_v c)^{-1}\beta g Q_0$$

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By using a Fourier transformation with respect to z, a Hankel transformation with respect to r, and a Laplace transformation with respect to t, the solution of problem (2.2) may be represented in the form

$$B = \frac{M}{4\pi^2 i} \iiint \exp(pt + ik_1 z) J_1(rs) s^2 \times \{[p + \nu(s^2 + k_1^2)]^2 (s^2 + k_1^2) + \omega_0^2 s^2 + f_0^2 k_1^2\}^{-1} ds\, dk_1\, dp \tag{2.3}$$

(J_1 is a first-order Bessel function of the first kind).

In (2.3) let us replace the variables as follows:

$$z = R\cos\theta, \quad r = R\sin\theta, \quad R^2 = z^2 + r^2$$
$$k_1 = k\cos\varphi, \quad s = k\sin\varphi, \quad dk_1 ds = k dk d\varphi \tag{2.4}$$

and carry out an inverse Laplace transformation taking account of (2.4)

$$B = M \int_0^{\pi/2}\int_0^{\infty} \exp(-\nu k^2 t) \sin k_1 z \frac{\sin\omega t}{\omega} J_1(rs) \cos^2\varphi\, dk d\varphi$$
$$\omega = (\omega_0^2 \cos^2\varphi + f_0^2 \sin^2\varphi)^{1/2}$$

Let us now substitute the expression for $J_1(rs)$ according to Bessel's integral formula

$$J_1(x) = \frac{2}{\pi}\int_0^{\pi/2} \sin(x\sin\Phi)\sin\Phi\, d\Phi$$

and integrate with respect to k. We obtain a new integral representation for the function $B(R, \theta, t)$

$$B = \frac{M}{\pi^2}\int_0^{\pi/2}\int_0^{\pi/2} \sin\omega t \cos^2\varphi \frac{\sin\Phi}{R^2\omega}\left(\frac{R^2}{4\nu t}\right)^{3/2} (B^+ + B^-)\, d\Phi\, d\varphi \tag{2.5}$$
$$B^\pm = m^\pm \exp\left[-\frac{(Rm^\pm)^2}{4\nu t}\right], \qquad m^\pm = \sin\Phi\cos\varphi\sin\theta \pm \sin\varphi\cos\theta$$

This representation turned out to be convenient for investigating flow in the regions $R \ll \sqrt{4\nu t}$ and $R \gg \sqrt{4\nu t}$ where simple asymptotic expressions can be obtained for $\psi(R, \theta, t)$.

Structure of the flow when $R \ll \sqrt{4\nu t}$ (the viscous region). In this case the exponents in (2.5) can be put to unity, which leads to the expression

$$B(R, \theta, t) = M^* \int_0^{\pi/2}\int_0^{\pi/2} \frac{\sin\omega t}{\omega} \cos^2\varphi \sin^2\Phi\, d\Phi\, d\varphi$$
$$M^* = M\frac{R\sin\theta}{(4\nu t)^{3/2}}$$

Using the stationary-phase method when $\omega_0 t \gg 1$, we obtain

$$\psi(R, \theta, t) \sim \sqrt{\frac{\pi}{2}} M^* \delta \frac{\sin(\omega_0 t - \pi/4)}{\sqrt{\omega_0 t}}, \quad f_0 \neq \omega_0 > 0 \tag{2.6}$$
$$\psi(R, \theta, t) \sim \sqrt{\frac{\pi}{2}} M^* \frac{\sin\omega_0 t}{\omega_0}, \quad f_0 = \omega_0 > 0$$
$$\delta = |f_0^2 - \omega_0^2|^{-1/2}, \quad M^* = \frac{(R\sin\theta)^2}{(4\nu t)^{3/2}}$$

It follows from relationships (2.6) that, in both cases, vibrations arise due to stable stratification with a slower decay of the vibrations in the case when $f_0 = \omega_0 > 0$. When there is neutral stratification ($\omega_0 = 0$), there are no vibrations if $f_0 > 0$ and the rising flow

$$\psi(R, \theta, t) \sim M^* f_0^{-1}, \quad f_0 t \gg 1 \tag{2.7}$$

occurs.

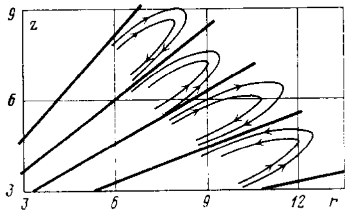

Fig.1

Structure of the flow when $R \gg \sqrt{4\nu t}$ (the non-viscous region). In this region the influence of viscous effects disappears and, after application of the method of steepest descent, representation (2.5) reduces to the expression

$$B(R,\theta,t) = \frac{M}{(R\sin\theta)^2}\int_0^{\theta}\frac{\cos\varphi\sin\omega t}{\omega(1-\operatorname{tg}^2\varphi\operatorname{tg}^{-2}\theta)^{1/2}}\,d\varphi \tag{2.8}$$

Here, by replacing the variable $\operatorname{tg}^2\varphi\operatorname{tg}^{-2}\theta = \sin\alpha$ and using the stationary-phase method when $\omega_0 t \gg 1$, we obtain, when $f_0 \neq \omega_0$

$$\psi = \psi_0 - \psi_\omega,\quad \psi_0 \sim \sqrt{\frac{2}{\pi}}\frac{M\delta}{R\sin\theta}\frac{\sin(\omega_0 t - \pi/4)}{\sqrt{\omega_0 t}} \tag{2.9}$$

$$\psi_\omega \sim \sqrt{\frac{2}{\pi}}\frac{M\delta\cos\theta}{R\sin\theta}\frac{\sin(\overline{\omega}t - \pi/4)}{\sqrt{\overline{\omega}t}},\quad \overline{\omega} = (\omega_0^2\cos^2\theta + f_0^2\sin^2\theta)^{1/2}$$

It is obvious from formulae (2.9) that, when $\omega_0 t \gg 1$, two harminocs: ψ_0 and ψ_ω, manifest themselves in the flow. The first harmonic with a frequency $\omega = \omega_0 = \text{const}$ is solely due to stratification while the second harmonic, with a frequency $\omega = \overline{\omega}$, reflects the anisotropy of the medium which arises due to stratification and rotation. This harmonic describes the stratification of the flow on vortices which, by arranging themselves one above the other, create a multilevel structure of non-stationary convective flows (see Fig.1 where $\psi = 0$ corresponds to the straight lines).

The nature of the flow changes abruptly when $f_0 = \omega_0$. Then, from (2.8), we have

$$\psi(R,\theta,t) \sim MR^{-1}\sin^2\theta\sin f_0 t \tag{2.10}$$

and, in this case, the motion of the medium is coherent in the whole of space, no stratification occurs and the multilevel structure unfurls into an oscillating annular rotation with a vortex.

It is obvious from (2.10) that, unlike the case when there is no stratification (2.7), gyroscopic waves are excited in the region $R \gg \sqrt{4\nu t}$ $(f_0 > 0)$.

3. Convection when $\nu \gg \varkappa \gg D$.

In the general case system (1.1) leads to the equation for $B(r,z,t)$

$$\Big[A_\varkappa A_D\Big(A_\nu^2 + f_0^2\frac{\partial^2}{\partial z^2}\Big) + \omega_0^2 A_\nu A_\varkappa\Big(\Delta - \frac{\partial^2}{\partial z^2}\Big)\Big]B = \tag{3.1}$$
$$MA_\nu A_D\delta(t)\,\delta(z)\frac{\partial}{\partial r}\delta(r)$$
$$A_\nu = \frac{\partial}{\partial t} - \nu\Delta,\quad A_\varkappa = \frac{\partial}{\partial t} - \varkappa\Delta,\quad A_D = \frac{\partial}{\partial t} - D\Delta$$

which is of the fourth order with respect to time and of the tenth order with respect to the spatial variables. The analytical investigation of Eq.(3.1) does not give rise to any fundamental difficulties but it is an enormous task and, for this reason, we shall consider a simpler case when $f_0 = 0$. Then, Eq.(3.1) together with the conditions for the boundedness of the perturbations leads to the problem

$$A_\varkappa\Big[A_\nu A_D\Delta + \omega_0^2\Big(\Delta - \frac{\partial^2}{\partial z^2}\Big)\Big]B = MA_D\delta(t)\,\delta(z)\frac{\partial}{\partial r}\delta(r) \tag{3.2}$$
$$r = 0,\infty,\quad z = \pm\infty,\quad B = 0$$

We note that, in this case, Eq.(3.2) is of a higher order than (2.2) which signifies the existence of additional properties for the convective flow (3.2). By applying integral transforms analgous to (2.2)-(2.4) to (3.2) and inverting the Laplace transform, we can represent the function $B(R,\theta,t)$ in the form

$$B = \frac{M}{\pi}\int_0^{\pi/2}\int_0^{\infty}(S_\varkappa + S_\nu)\cos(kz\cos\varphi)\,J_1(kr\sin\varphi)\,k\cos^2\varphi\,dk\,d\varphi \tag{3.3}$$
$$S_\varkappa = \frac{(D-\varkappa)k^2\omega_0^{-2}}{\cos^2\varphi - l_3^4k^4}\exp(-\varkappa k^2 t),\quad S_\nu = \frac{(S^+ + S^-)}{2}$$
$$S^{\pm} = (\pm\omega)^{-1}\exp(-\nu_0 k^2 t \pm \omega t)\frac{(\nu_0 - D)k^2 \pm \omega}{(\nu_0 - \varkappa)k^2 \pm \omega}$$
$$\omega = [\omega_0^2\cos^2\varphi - (\nu_0 - D)^2k^4]^{1/2},\quad \nu_0 = \frac{\nu + D}{2}$$

$$l_3=\left[\frac{(\nu-\varkappa)(\varkappa-D)}{\omega_0^2}\right]^{1/4}\sim\left(\frac{\nu\varkappa}{\Gamma g}\right)^{1/4},\quad \nu\gg\varkappa\gg D \tag{3.4}$$

Depending on the physical processes determining the flow field (3.2), (3.3), we separate out the following space-time regions and regimes of motions.

Gravitational vibrations and jet stream. We use the method of steepest descent and stationary-phase method and, when $R\ll\sqrt{4Dt}$, and $\omega_0 t\gg 1$ in this region, we have

$$\begin{aligned}&\psi=\psi_\nu+\psi_\varkappa\\ &\psi_\nu\sim M\sqrt{\frac{\pi}{2}}\,\frac{(R\sin\theta)^2}{(4\nu_0 t)^{3/2}}\,\frac{\sin(\omega_0 t-\pi/4)}{\sqrt{\omega_0 t}}\\ &\psi_\varkappa\sim M\omega_0^{-2}(\varkappa-D)\frac{(R\sin\theta)^2}{(4\varkappa t)^{5/2}}\end{aligned} \tag{3.5}$$

The function ψ_ν is determined by the contribution of the spectrum $S_\varkappa(k)$ in the neighbourhood of zero and represents vibrations which are essentially analogous to (2.6) when $f_0=0$.

The function $\psi_\varkappa$ is determined by the spectrum $S_\varkappa(k)$ and describes a jet stream which decays monotonically with time and only arises in the case when $\varkappa\neq D$. This mode is dominant in (3.5) in the time interval $(\nu/\varkappa)\,\omega_0^{-1}\ll t\ll(\nu/\varkappa)^3\,\omega_0^{-1}$.

Dissipative structures and non-stationary vortices. When $(\nu-\varkappa)(\varkappa-D)>0$, the simple poles of the spectra $S_\nu(k)$ and $S_\varkappa(k)$ are located on the real axis of wave numbers k. If $\sqrt{4Dt}\ll R\ll\sqrt{4\varkappa t}$, and $(\nu/\varkappa)^{1/2}\omega_0^{-1}\ll t\ll(\nu/D)^{1/2}\,\omega_0^{-1}$, the spectrum $S_\nu(k)$ in the neighbourhood of a pole is barely deformed with the passage of time, that is, it is quasistationary. By calculating the contribution of the pole of $S_\nu(k)$ and using the stationary-phase method, we have the following asymptotic expression for ψ:

$$\begin{aligned}&\psi\sim M(\omega_0\nu_0)^{-1/2}f(\theta)\sin\left[Rl_3^{-1}m(\theta)\right],\quad R\gg l_3\\ &f(\theta)=\sin^{1/2}\theta\,\{\sin(\varphi_\nu-\theta)\left[\sin^{-2}(\varphi_\nu-\theta)+{}^1/_2\cos^{-2}\varphi_\nu\right]\}^{-1/2}\\ &m(\theta)=\cos^{1/2}\varphi_\nu\sin(\varphi_\nu-\theta),\quad \varphi_\nu(\theta)={}^1/_2\,[\theta+\\ &\quad\arccos(-{}^1/_3\cos\theta)]\end{aligned} \tag{3.6}$$

The asymptotic formula (3.6) shows that quasistationary cells with a characteristic size l_3, which is determined by formula (3.4), arise in the flow.

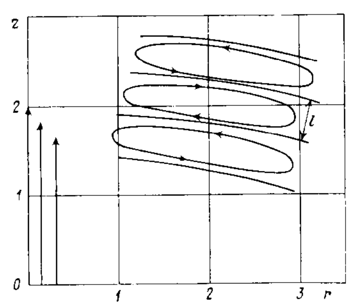

Fig.2

The formation of these cells is attributable to the combined action of stratification and the significant difference between the exchange coefficients $(\nu\gg\varkappa\gg D)$ since a slower transfer of salinity compared with the transfer of heat causes particles which have been cooled to settle at their own level of neutral buoyancy which leads to the formation of a local system of quasistationary disipative structures. The structure of the flow (2.5), (3.6) is depicted qualitatively in Fig.2.

In the region $R\gg\sqrt{4\nu t}$ formula (2.9) holds (when $f_0=0$). This formula describes a non-stationary multilevel system of vortices (Fig.1) which, in general, is independent of the dissipative characteristics of the medium. Here the flow is created both by the pressure field which is formed by the whole region of flow as well as by perturbations in the salinity c' which are generated by the source Γv_z (1.1) /2/ which has been induced.

4. Stationary convection from a point of heat for arbitrary $\nu,\varkappa,D>0$.

In this case system (1.1) reduces to the problem

$$\begin{aligned}&\left[\nu_0^2\Delta^3+\omega_0^2\left(\Delta-\frac{\partial^2}{\partial z^2}\right)+f_0^2\frac{\partial^2}{\partial z^2}\right]B=M\delta(z)\frac{\partial}{\partial r}\delta(r)\\ &r=0,\infty,\quad z=\pm\infty,\quad B=0\\ &\nu_0=\nu\left(\frac{\varkappa}{\nu}\right)^{1/2},\quad \omega_0=\left(\Gamma g\frac{\nu}{D}\frac{\varkappa}{\nu}\right)^{1/2}\end{aligned} \tag{4.1}$$

By carrying out the procedure described by (2.3)-(2.5), we have the following integral representation for $B(R,0)$:

$$B=B^++B^- \tag{4.2}$$

$$B^{\pm}=2M(\nu_0\omega_0)^{-1}\int_0^{\pi/2}\int_0^{\pi/2}\frac{\cos^2\varphi\sin\Phi}{\omega(\eta,\varphi)}\exp\left[\frac{R}{l}|m^{\pm}|\omega\right]\times$$
$$\sin\left[\frac{R}{l}|m^{\pm}|\omega\right]d\Phi\,d\varphi$$
$$\omega(\eta,\varphi)=\cos^2\varphi+\eta^2\sin^2\varphi,\quad \eta=f_0\omega_0^{-1},\quad l=(2\nu_0\omega_0^{-1})^{1/2}$$

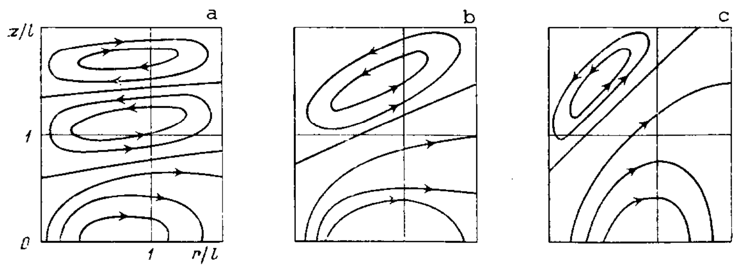

Fig.3

Asymptotic integration of (4.2) is possible in the regions $R\ll l$ and $R\gg l$.

Flow close to the source and the limits of applicability of the linear solution. Let us put the parameter $R/l\ll 1$ in (4.2) and confine ourselves to the first approximation in the expansion of the functions B^+ and B^- with respect to this small parameter. Then, the determination of $B(R,\theta)$ (and, correspondingly, of ψ) reduces to looking up tabulated integrals. We obtain the following expressions for the stream function and the vertical velocity in this region:

$$\psi=\psi^*f(\eta) \tag{4.3}$$
$$\psi^*=M\omega_0^{-1/2}\nu_0^{-3/2}(R\sin\theta)^2+O[(R/l)^3],\quad v_z=v_z^*f(\eta)$$

According to /3/, the functions ψ^* and v_z^* are determined when there is no rotation ($f_0=0$). The function $f(\eta)$ is expressed in terms of a Gauss hypergeometric function. As η increases, the magnitude of $f(\eta)$ decreases monotonically from a maximum value of unity when $\eta=0$ to zero when $\eta=\infty$.

Formulae (4.3) reflect the nature of the stabilizing action of rotation ($f_0>0$) on convection. Consequently, the limits of applicability of the linear solution, which were obtained in /3/ in the case where there is no rotation, are sufficient.

Structure of the flow remote from the source ($R\gg l$). In this case, on integrating with respect to Φ, the neighbourhood of the point $\Phi_0=\arcsin(\operatorname{tg}\varphi/\operatorname{tg}\theta)$ makes the main contribution to the integral B^-. By integrating with respect to Φ in (4.2) and then with respect to φ, we obtain the expression

$$\psi\sim M\pi^2\nu/\varkappa^{-1}(\Gamma\nu/D^{-1}R\cos\theta)^{-1}(1+\eta^2\operatorname{tg}^2\theta)^{-1/2},\quad r\gg z\gg l \tag{4.4}$$

which determines the nature of the flow in the main cell close to the radial surface $r\gg z\gg l$.

In the region $z\gg r\gg l$ the flow is vertically stratified into a number of cells which are described in /3/. Rotation leads to an increase in the angle of inclination of the boundaries of the cells to the radial plane and, at the same time, the width of the main cell of (4.4) increases. Hence, as the rotation increases (as η increases) the cells are tilted and displaced into a region adjacent to the vertical axis, i.e. the horizontal orientation of the cells gradually changes to a vertical orientation. The effect of rotation on the structure of the flow is shown in Fig.3 using the stream lines obtained from a numerical calculation of (4.2) when $\eta=0.1$ (a), $\eta=0.5$ (b), and $\eta=1$ (c). These cells, like Bénard cells, are dissipative structures, the stable stationary state of which is maintained due to the combined action of buoyancy (when there is stratification), the pressure field, thermal diffusivity, viscosity and diffusion.

The azimuthal velocity v_φ. System (1.1) leads to the following problem for the determination of v_φ in the stationary case:

$$\left[\nu_0^2\Delta^3+\omega_0^2\left(\Delta-\frac{\partial^2}{\partial z^2}\right)+f_0^2\frac{\partial^2}{\partial z^2}\right]\Delta v_\varphi=Mf_0\nu^{-1}\delta(z)\frac{\partial}{\partial r}\delta(r) \tag{4.5}$$
$$r=0,\infty,\quad z=\pm\infty:\quad v_\varphi=0$$

From (4.5), we have the following integral representation:

$$v_\varphi=M\pi f_0\nu^{-3/2}\omega_0^{-1/2}\eta(A^++A^-) \tag{4.6}$$

$$A^{\pm} = \int_0^{\pi/2}\int_0^{\pi/2} \frac{\sin\varphi\cos^2\varphi\sin\Phi}{\omega^{3/2}(\eta,\varphi)} \exp R^{\pm}(\cos R^{\pm} + \sin R^{\pm})\, d\Phi\, d\varphi$$
$$R^{\pm} = Rl^{-1}\omega^{1/2}(\eta,\varphi)\,|m^{\pm}|$$

a) When $R \ll l$, we have from (4.6)

$$v_\varphi \sim M\nu_0^{-3/2}\omega_0^{-1/2}R^2l^{-2}\sin\theta\cos\theta F(\eta) \tag{4.7}$$

where $F(\eta)$ is a monotonically increasing function, $F(\eta) \sim \eta$ when $\eta \ll 1$ and $F(\eta) \sim \sqrt{\eta}$ when $\eta \gg 1$.

b) When $R \gg l$ within the bounds of the region of the main cell $r \gg z \gg l$ (4.4), for v_φ from (4.6) we have the asymptotic form

$$-v_\varphi \sim M2^{-1/2}\nu_0^{-3/2}\omega_0^{-1/2}R^{-1}l\,\mathrm{ctg}(\theta)\,P(\eta) \tag{4.8}$$

where $P(\eta)$ is well approximated by the function $P(\eta) \sim \eta(1+\eta^2)^{-1}$. The slower nature of the decrease in the azimuthal velocity (4.8) in this region than in the case of the radial and vertical velocities (4.4) should be noted. Formulae (4.7) and (4.8) enable one to establish the dependence of the intensity of the twisting $v_\varphi(r, z)$ on the radius when $z = \mathrm{const}$ in the main cell. When $R \gg l$, the motion is close to the rotation of a solid body ($v_\varphi \sim r$) while, when $R \gg l$, it is close to a potential vortex ($v_\varphi \sim 1/r$).

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DEPENDENCE OF THE DISPERSION CURVES OF INTERNAL WAVES OF A STRATIFIED OCEAN ON THE VAISALA-BRENT FREQUENCY*

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Under the condition that the minimum of the Vaisala-Brent frequency (VBF) is greater than the Coriolis parameter, a parametric form of the dispersion curves of the internal gravitational waves in an ocean of constant depth with continuously variable VBF is obtained. This form is used when obtaining estimates of the dc displacements as a function of the VBF displacement and when isolating the VBF which admit of a unique restoration from a sequence of dispersion curves.

1. Formulation of the problem. We consider a horizontal continuously stratified ocean of constant depth $\mathbf{H}$. Its upper surface is the x/y plane, and the z axis is directed vertically upwards. The dispersion curves of the internal gravitational waves are found /1/ as the eigenvalues $\omega^2 = \omega_n^2(k^2)$ of the boundary value problem

$$W'' - \frac{\mu(z)}{g}W' + \frac{\mu(z)-\omega^2}{\omega^2 - f^2}k^2W = 0, \quad W(-\mathbf{H}) = W(0) = 0 \tag{1.1}$$

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